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culturist, from the high price of all the compounds of cyanogen.

Mr. George Yeates read a paper containing the results of a Meteorological Journal for the year 1843.—*See Appendix V.*

The Rev. H. Lloyd communicated a letter written many years ago to his father, the late Dr. Lloyd, Provost of Trinity College, by Mr. Mac Cullagh, who was then a Fellowship-Candidate in the College. It relates principally to a mechanical theory (that of the rotation of a solid body) which Mr. Mac Cullagh was occupied with at that period, and which he had occasion to allude to at the last meeting of the Academy. The following is an extract from the letter. The beginning and the date are wanting.

“THEOREM I.—If a rigid body, not acted on by any extraneous forces, revolve round a fixed point O, and if an ellipsoid be described having its semiaxes in the direction of the principal axes passing through O, and equal to the radii of gyration round them; then a perpendicular to the invariable plane being raised from O to meet the surface of the ellipsoid in I, the line OI (which is fixed in space, as the ellipsoid revolves with the body) will be of a constant length during the motion; and a perpendicular from O upon the plane which touches the surface at I, will always be the axis of rotation, and will vary inversely as the angular velocity.

“*Corollaries.*

“1. Since every radius which is nearly equal to the greatest or least semiaxis of an ellipsoid must lie near that semiaxis, it appears that if, in the beginning of the motion, the point I be near the vertex of either of these semiaxes, it will always be near it, since OI remains constant; and therefore, by the preceding construction, the axis of rotation will always remain near the same semiaxis. Hence the rotation about the axes of greatest and least moment in any body is stable. The rotation about the axis of mean moment is unstable, because the radii of an ellipsoid, which are nearly equal to the mean semiaxis, do not all lie near that semiaxis.

“These things are evident from considering the trace of the constant line OI on the surface of the ellipsoid, and observing that, in general, its projection on the plane of the greatest and least axes is a hyperbola, and its projections on the other two planes ellipses.

“2. But if OI be equal to the mean semiaxis b (a and c being the greatest and least) it will always intersect the body in the same plane, and the ellipsoid in a circle. For there are two circular sections through the mean axis; and therefore if the point I be at any instant in either of them, it will remain in it during the motion. It would be easy to show also, that the axis of rotation, connected with OI by the construction in the proposition, will in this case always remain in a given plane within the body.

“3. If two axes (or moments) of the ellipsoid are equal, OI will describe in the body a conical surface round the third, and the axis of rotation will always be in the same plane with OI and that third axis, and these three lines will make constant angles with each other; also the perpendicular on the tangent plane, and therefore the angular velocity, will be constant. These things are evident merely from considering that the ellipsoid becomes one of revolution.

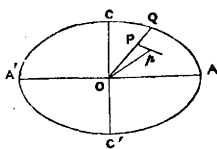
“4. Whatever be the forces applied to the body, the varying plane of the maximum of areas and the axis of rotation are always connected by the construction in the proposition. But the angular velocity is no longer inversely as the perpendicular.

“To find the axis about which a body restrained by a fixed point O , and acted on by given forces, will begin to revolve, is usually considered a problem of great complexity. But it may be elegantly solved by means of the ellipsoid described above. Reduce the given forces to a single one through the fixed point O , and a pair; raise a perpendicular to the plane of the pair from O to meet the ellipsoid in I ; a perpendicular from O to the tangent plane at I will be the initial axis of rotation.

“The construction is true whether the forces that set the body in motion be impulses or pressures. If they be impulses, and no external forces subsequently act on the body, the axis of rotation will vary its position both in the body and in space; its course in the body is determined by the preceding theorem, as well as the variation of the angular velocity. The motion of the body in space depends mainly on the two following theorems and the rectification of the ellipse.

Let the principal axes of the ellipsoid (or a, b, c) be OA, OB, OC .

“THEOREM II.—If a perpendicular IP be let fall from I on any of the principal planes (as AOC), the *areolar* velocity of P round O will be proportional to the perpendicular IP .



“By areolar velocity I mean the increment of the area (as POp) divided by the increment of the time when taken indefinitely small.

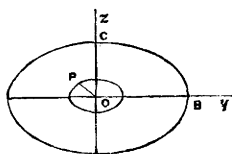
“Since $IP = \sqrt{(OI^2 - OP^2)}$, the position of OI , and therefore of the axis of rotation at any given time, may be determined from this theorem by the method of quadratures; and it may be reduced to the rectification of the ellipse.

“THEOREM III.—Let a plane passing through the fixed line OI and any of the principal axes (as OB), intersect the plane AOC in OQ , and the invariable plane (to which OI is perpendicular) in a straight line which may be called OR ; the angular velocity of OR is inversely as the square of OQ ; and hence if OR be always taken equal to OQ , the point R will describe in the invariable plane areas proportional to the times.

“Since OQ is known at any time by the preceding proposition, the position of OR at any time will be known from this by the method of quadratures. Also the inclination of the plane AOC to the invariable plane is known, since it is equal to the angle OIP . Hence the position of the body at any instant is completely determined.

“For an application of the theorems let us take the following problem:—The body revolving round a line indefinitely near the greatest or least of the principal axes, to find the time of an oscillation. By the time of an oscillation I mean that in which OI , and consequently the axis of rotation, returns to the same position within the body.

“Let the axis of rotation be indefinitely near OA . Then x, y, z being the coordinates of I , we have $x^2 + y^2 + z^2 = OI^2 = k^2$, and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Therefore



$$\left(\frac{1}{b^2} - \frac{1}{a^2}\right)y^2 + \left(\frac{1}{c^2} - \frac{1}{a^2}\right)z^2 = 1 - \frac{k^2}{a^2}.$$

Hence the locus of P is an ellipse whose semiaxes a' and b' are

$$\frac{b\sqrt{a^2 - k^2}}{\sqrt{a^2 - b^2}} \text{ and } \frac{c\sqrt{a^2 - k^2}}{\sqrt{a^2 - c^2}};$$

and therefore its area

$$\pi a'b' = \frac{\pi bc(a^2 - k^2)}{\sqrt{(a^2 - b^2)(a^2 - c^2)}}.$$

“But I ought to have mentioned, as part of Theorem II., the method of determining in general the areolar velocity of P. Let the angular velocity multiplied by the cosines of the angles which the axis of rotation makes with OA, OB, OC be denoted (as is usual) by p, q, r . These have constant ratios to the perpendiculars, as IP, drawn in that proposition; and in that case the areolar velocity of P is equal to $\frac{1}{2}(b^2 - k^2)q$: and similarly when AOC is any of the other principal planes. Hence, in the present instance, the areolar velocity = $\frac{1}{2}(a^2 - k^2)p$; and therefore the time of an oscillation

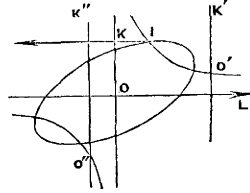
$$= \frac{2\pi bc}{p\sqrt{(a^2 - b^2)(a^2 - c^2)}},$$

from the above value of the area of the ellipse, and observing that, since OI is indefinitely near OA, IP and therefore p may be regarded as constant.

“If τ denote the time of one revolution of the body round its axis, then ultimately $\tau = \frac{2\pi}{p}$, and therefore the time of an oscillation is to the time of a revolution as the rectangle under the semiaxes of the section BOC is to the rectangle under the eccentricities of the other two sections. A similar theorem holds when the body revolves round a line indefinitely near the least principal axis. The times of small oscillations of different magnitudes are equal, as in the pendulum.

“Many particular consequences might be deduced from what has been said; but it will be better to mention some new theorems about moments of inertia and centrifugal forces.

"The forces resulting from the centrifugal forces of a body of any figure, revolving round an axis passing through a fixed point O, may be found elegantly by the ellipsoid of which we have already made so much use. Let a plane at right angles to the given axis OK, and cutting it in K, touch the ellipsoid in I; the centrifugal forces will be reduced to a pair whose moment is $OK \times KI \times \omega^2$ (ω being the angular velocity), whose plane is OKI, and direction as marked in the figure; and a single force equal to $Mp\omega^2$, p being the distance of the centre of gravity from OK.



"If OK pass through the centre of gravity, there remains only the pair. The perpendicular OK is the radius of gyration for the axis OK. Particular consequences of these things are numerous.

"Any line, taken at random in a body, may not be a principal axis. All the principal axes parallel to a given line lie in the same plane; and the points of their lengths which must be fixed, in order that they may be principal axes, will lie in a hyperbola. Suppose in this case the point O (preceding fig.) to be the centre of gravity, and OK to be parallel to the given line, and describe through I an equilateral hyperbola whose asymptotes are OK and OL; then all the principal axes, as O'K', O''K'', parallel to OK, lie in the plane OKI, and the points O', O'' of their lengths, which must be fixed, are at their intersections with the hyperbola.

"By means of theorems of this nature, all of which are proved geometrically, without any calculation, I have been able to give a complete geometrical solution of the problem of the motion of a solid body not acted on by any forces. If it be acted on by given forces, the differential equations in A, B, C and p, q, r , which are given by all the writers on mechanics, are direct consequences of the first principles, without the intervention of any calculation.

* * * * *

"Another thing with which I had occupied myself is the attrac-

* Multiplied by the mass of the body, or by M.

tion of ellipsoids. Having written out a simple demonstration of a very elegant known theorem relating thereto, I shall subjoin it separately.

* * * * *

“ The proposition above alluded to depends on the following theorem, which may be very simply proved :—If from the extremities A, B, C of the semiaxes of an ellipsoid whose centre is O, there be drawn three parallel chords A α , B β , C γ , meeting the surface of the ellipsoid in α , β , γ ; and if a perpendicular from α on OA meet it in r , a perpendicular from β on OB meet OB in s , and a third from γ on OC meet OC in t ; then will

$$\frac{Ar}{AA'} + \frac{Bs}{BB'} + \frac{Ct}{CC'} = 1.$$

AA', BB', CC' are the whole axes.

“ The proposition itself is this :—If the particles of a homogeneous ellipsoid attract inversely as the square of the distance, and if a , b , c be the semiaxes, and A_0 , B_0 , C_0 its attractions on points placed at their vertices, then will

$$\frac{A_0}{a} + \frac{B_0}{b} + \frac{C_0}{c} = 4\pi.$$

“ The attractions are here, as is usual, represented by lines; the attraction of an indefinitely small part of the solid being represented by its *volume* divided by the square of its distance from the attracted point.

“ The attraction being thus measured, it is evident that if from the vertex of a pyramid, whose transverse sections are indefinitely small, as a centre, with any radius, a sphere be described whose surface is penetrated by that of the pyramid, the attraction of the pyramid on a point at its vertex will be to its length, as the intercepted surface of the sphere is to the square of its radius.

“ Let A, B, C be the three vertices of the axes, and from them let parallel chords A α , B β , C γ be drawn; from whose extremities α , β , γ let perpendiculars be let fall on the respective axes, meeting them in r , s , t ; then by the preceding theorem

$$\frac{Ar}{AA'} + \frac{Bs}{BB'} + \frac{Ct}{CC'} = 1.$$

Let now two other chords be drawn from A making with $A\alpha$ very small angles, so as to form with it the edges of a very small pyramid, and let other chords parallel to them be drawn from B and C, forming also with $B\beta$ and $C\gamma$ the edges of two other small pyramids. Imagine a sphere fixed in space, from whose centre are drawn three lines parallel to the three chords drawn from A, or from B, or from C, and conceive the surface of the pyramid, of which they are the edges, to penetrate that of the sphere; then will the attractions of the three pyramids, reduced each to the direction of the axis passing through its vertex, be to Ar , Bs , Ct as the intercepted surface of the sphere to the square of its radius; and therefore the sum of each of those attractions, divided respectively by AA' , BB' , CC' , will be to unity in the same ratio. Conceive pyramids thus related to be multiplied indefinitely, and the spheroid will be exhausted at once from each of the three points A, B, C, while half the surface of the sphere is exhausted by the parallels drawn from its centre. Hence it appears that the sum of the whole attractions at A, B, C, divided respectively by AA' , BB' , CC' , is to unity as the surface of a hemisphere to the square of its radius, or as 2π to 1; and therefore

$$\frac{A_0}{a} + \frac{B_0}{b} + \frac{C_0}{c} = 4\pi.$$

January 22.

REV. JAMES H. TODD, D. D., Vice-President, in the Chair.

Mr. W. H. Hudson exhibited specimens of Irish books, now in the course of publication in Cork, which are lithographed. He described the advantages which that process presents over types, for composition in the Irish character.

Dr. Kane read a paper on the Chemical Composition of the different kinds of fuel found in Ireland.